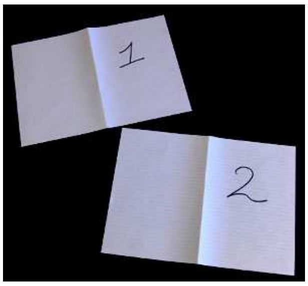
Exam NANP1-10 Physics Laboratory 1: Data and error analysis
3 February 2017 (14:00 - 16:00)

Please note:

- **DO NOT OPEN THE EXAM BEFORE YOU ARE TOLD TO DO SO!**
- This exam consists of **6 exercises** on 5 pages.
- **Make each exercise on a separate sheet of paper!**



- **Write your name and student number on each sheet of paper!**
- Write clearly, using a pen (not a pencil).
- A simple scientific calculator is allowed during the exam, but a graphing calculator is not permitted.
- When finished, hand in these exercises together with your own answers.

Points per exercise

Exercise		total	Exercise		total
1a,b,c,d,e	1 each		4		4
1		5	5a	2	
			5b	2	
2a	1		5c	2	
2b	5		5d	2	
2c	1		5e	1	
2		7	5		9
3a	2		6a	2	
3b	2		6b	2	
3		4	6c	3	
4a	2		6d	2	
4b	2		6		9

Exam mark = (total of points / 4.2) + 1.0

Exercise 1 *(5 points)*

In parts a) - d), **rewrite** the following results, using the correct notation (proper accuracy and number of significant digits):

a) $v = 7.438$ km/s $\pm$ 213 m/s

b) $\lambda = 384.2$ nm $\pm$ 0.032 μm

c) $T = 5764.15$ K $\pm$ 0.24 mK

d) $p = 2.891$ MPa $\pm$ 12.4 kPa

The decay of a sample of radioactive material is measured and an *average* rate of 4.0 decays per second is observed.

e) **Multiple choice question:** what is the probability P of observing *exactly* 2 decays in 1 second?
A: $P = 75\%$
B: $P = 15\%$
C: $P = 50\%$
D: $P = 25\%$

Your answer does not have to contain more than a single letter {A, B, C, D}. Any arguments or calculations given will be ignored. Given:

$$f(k) = \frac{\lambda^k}{k!} e^{-\lambda}$$

Exercise 2 *(7 points)*

The impedance Z ("resistance" to alternating current [AC]) of an electrical circuit containing a resistor and a capacitor in series is given by:

$$Z = \sqrt{R^2 + \left(\frac{1}{2\pi f C}\right)^2}$$

with R the resistance of the resistor, C the capacitance of the capacitor and f the frequency with which the current alternates. The following values are measured independently: $f = 50.00$ Hz (negligible error), $R = 5.6$ kΩ $\pm$ 5% and $C = 470 \pm 33$ nF.

a) **Calculate** the impedance Z.

b) **Calculate** the relative error and the absolute error in Z.

c) **Write** the final result in the correct notation: $Z = ... \pm ...$ ..

Exercise 3 *(4 points)*

The mass of a small object is determined using three different methods. The results are: $m_1 = 37.3 \pm 0.2$ g, $m_2 = 37.4 \pm 0.1$ g, $m_3 = 37.2 \pm 0.3$ g. The given errors represent the standard deviation.

a) **Calculate** the weighted average mass m_{avg}.

b) **Calculate** the error Δm of m_{avg}.

Exercise 4 *(4 points)*

The density ρ of diamond has been determined from measurements on a very large number of individual diamonds. The measurement results follow a Gaussian distribution, with $\rho = 3.515 \pm 0.015$ g/cm^3. Interpret the given error as the standard deviation.

a) **Calculate** the probability of finding an individual diamond for which the density is in the range $3.503 \leq \rho \leq 3.536$ g/cm^3.

b) **Calculate** the probability of finding an individual diamond for which $\rho \leq 3.497$ g/cm^3.

If necessary, use table 2 (on the last page) and the following formulae:

$$N(y) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{y^2}{2}\right) \qquad \text{and} \qquad F(z) = \int_0^z N(y)dy$$

Exercise 5 *(9 points)*

The temperature T inside a flask with evaporating liquid helium is measured 5 times. The observations are: $T = 4.2$ K, 4.1 K, 4.4 K, 4.5 K and 4.0 K. The measurements follow a Gaussian distribution.

a) **Calculate** the best estimate for the average temperature.

b) **Calculate** the best estimate s for the standard deviation of these measurements.

c) **Calculate** the error s_m in the best estimate for the average temperature calculated in part a).

d) The measurement of the temperature T is repeated 175 times *extra*. **By which factor** (that is, how many times) is the error calculated in part c) now reduced?

e) **Is this** [the method used in part d)] an efficient way to reduce the error in the final result? **Explain** why.

Exercise 6 *(9 points)*

x	$y \pm \Delta y$
-2	5 ± 2
-1	2 ± 3
1	1 ± 1
3	-2 ± 2

Table 1: Observations for exercise 6.

A series of 4 observations is given in table 1 above. The error in x is negligible. A straight line $y = ax + b$ is fitted to these observations (with equal weights for all data points).

The following formulae are given for fitting data to a straight line $y = ax + b$:

$$a = \frac{N \sum x_i y_i - \sum x_i \sum y_i}{N \sum x_i^2 - \left(\sum x_i\right)^2},$$

$$b = \frac{\sum y_i \sum x_i^2 - \sum x_i \sum x_i y_i}{N \sum x_i^2 - \left(\sum x_i\right)^2},$$

$$\Delta a = \sqrt{\left(\frac{1}{\sum x_i^2 - N\overline{x}^2}\right) \frac{\sum r_i^2}{N-2}},$$

$$\Delta b = \sqrt{\left(\frac{1}{N} + \frac{\overline{x}^2}{\sum x_i^2 - N\overline{x}^2}\right) \frac{\sum r_i^2}{N-2}}.$$

a) **Calculate** the best estimate for a and b using the method of least squares.

If you do not have the answer to part a), use $a = -1.5$ and $b = 2$ for the rest of the exercise.

b) **Calculate** the errors in a and b.

The student who has carried out the experiment wants to use the chi-square test to check whether the linear fit is acceptable. She decides to use the 1% - 99% acceptance requirement.

c) **Calculate** χ^2.

d) **Indicate** whether the linear fit is acceptable or not. If necessary, use table 3 (on the last page).

Tables

z	$F(z)$	z	$F(z)$	z	$F(z)$	z	$F(z)$
0.0	0.0000	1.0	0.3413	2.0	0.4772	3.0	0.4987
0.1	0.0398	1.1	0.3643	2.1	0.4821	3.1	0.4990
0.2	0.0793	1.2	0.3849	2.2	0.4861	3.2	0.4993
0.3	0.1179	1.3	0.4032	2.3	0.4893	3.3	0.4995
0.4	0.1554	1.4	0.4192	2.4	0.4918	3.4	0.4997
0.5	0.1915	1.5	0.4332	2.5	0.4938	3.5	0.4998
0.6	0.2258	1.6	0.4452	2.6	0.4953	3.6	0.4998
0.7	0.2580	1.7	0.4554	2.7	0.4965	3.7	0.4999
0.8	0.2881	1.8	0.4641	2.8	0.4974	3.8	0.4999
0.9	0.3159	1.9	0.4713	2.9	0.4981	3.9	0.5000

Table 2: Numerical values of the Gaussian integral function $F(z)$.

$F =$ ν	0.01	0.10	0.50	0.90	0.99
1	0.000	0.016	0.455	2.706	6.635
2	0.020	0.211	1.386	4.605	9.210
3	0.115	0.584	2.366	6.251	11.35
4	0.297	1.064	3.357	7.779	13.28
5	0.554	1.610	4.351	9.236	15.09

Table 3: Cumulative χ^2 distribution $F(\chi^2|\nu)$.